Perturbative and Non-perturbative Corrections to $B \to D^{(*)} l\nu$*

Andreas S. Kronfeld[a]

[a]Theoretical Physics Group, Fermi National Accelerator Laboratory, Batavia, Illinois, U.S.A.

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It is shown that certain double ratios introduced for computing semileptonic form factors are accurate to order $1/m_Q^2$, even when the action and current are accurate to order $1/m_Q$.

1. INTRODUCTION

The semileptonic decays $B \to Dl\nu$ and $B \to D^*l\nu$ are crucial to the determination of the entry $|V_{cb}|$ in the CKM matrix. Experiments measuring the differential rate of these processes require theoretical input to extract $|V_{cb}|$, namely the form factors of the hadronic transition.

To calculate the form factors we have introduced several double ratios [1,2]

$$R_+ = \frac{\langle D|V_0|B\rangle\langle B|V_0|D\rangle}{\langle D|V_0|D\rangle\langle B|V_0|B\rangle} = \rho^{-2}_{V_0^{cb}}|h_+|^2; \tag{1}$$

a similar ratio R_1 defined by replacing the pseudoscalars B and D with vectors B^* and D^*;

$$R_- = \frac{\langle D|V_i|B\rangle}{\langle D|V_0|B\rangle}\frac{\langle D|V_0|D\rangle}{\langle D|V_i|D\rangle} = \rho^{-1}_{V_i^{cb}}\left[1 - \frac{h_-}{h_+}\right]; \tag{2}$$

and

$$\begin{aligned} R_A &= \frac{\langle D^*|\epsilon\cdot A|B\rangle\langle B^*|\epsilon\cdot A|D\rangle}{\langle D^*|\epsilon\cdot A|D\rangle\langle B^*|\epsilon\cdot A|B\rangle} \\ &= \rho^{-2}_{A^{cb}}\frac{h_{A_1}^{B\to D^*}h_{A_1}^{D\to B^*}}{h_{A_1}^{D\to D^*}h_{A_1}^{B\to B^*}}. \end{aligned} \tag{3}$$

The ratios R_+, R_1, and R_A are defined at zero recoil; the ratio R_- is defined in the limit of zero recoil. The ρ are matching factors, needed to patch radiative corrections from short distances.

The ratios R_+ and R_- directly give the form factors h_+ and h_-, which together form the hadronic amplitude for $B \to Dl\nu$. Information from R_+, R_1, and R_A must be extracted from their heavy quark expansions to obtain $h_{A_1}^{B\to D^*}$, the hadronic amplitude for $B \to D^*l\nu$ [2].

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In the limit of degenerate heavy quarks and in the infinite mass limit, all four double ratios are equal to one. Thus, one essentially computes the deviation of the ratios from one, and the statistical and systematic uncertainties are a fraction of $R-1$, not of R. This makes it possible to extract the $1/m_Q$ correction to h_- and the $1/m_Q^2$ corrections to h_-, h_+, h_1, and h_{A_1}, provided the action and currents are accurate enough. The aim of this paper is to explain a remarkable result: the double ratios yield the $1/m_Q^2$ corrections when the action and currents used to compute them are tuned only through order $1/m_Q$.

2. POWER CORRECTIONS

Properties of heavy quark states calculated with Wilson fermions can be interpreted by appealing to a non-relativistic effective theory, which provides a "factorization" of short-distance from long-distance physics [3]. As in any effective theory, the effects of short distances (here a, m_b^{-1}, and m_c^{-1}) are lumped into coefficients, while the effects of long distances (here $\Lambda_{\mathrm{QCD}}^{-1}$) are generated by local operators.

To deduce the operators of this effective theory, one can start by thinking about symmetries. The action for Wilson fermions can be written

$$S = \sum_x \bar\psi_x\psi_x - \kappa\sum_{x,y}\bar\psi_x M_{xy}\psi_y, \tag{4}$$

where κ is the hopping parameter and the hopping matrix M_{xy} may include a clover term and further improvement terms. The heavy-quark limit corresponds to small κ, and as $\kappa \to 0$ the lattice action (4) obviously acquires the spin and flavor symmetries [4] of continuum QCD in the

limit $\Lambda_{\rm QCD}/m_Q \to 0$. As long as $\Lambda_{\rm QCD}/m_Q \ll 1$, Green functions calculated from (4) are given by the static limit plus small corrections, no matter what value $m_Q a$ takes.

Thus, just as continuum QCD can be described by a heavy-quark effective theory (HQET), lattice QCD can be described by a modified HQET, as long as the relevant physical momenta $\boldsymbol{p}$ satisfy

$$|\boldsymbol{p}| \ll m_Q, \quad |\boldsymbol{p}| \ll 1/a. \tag{5}$$

The operators of the HQET for lattice QCD are the same as always, but the coefficients are different: they depend on ratios of short distances: am_b, am_c, and m_b/m_c. The operators are sensitive to the scale $\Lambda_{\rm QCD}$. Heavy-light matrix elements may have lattice artifacts, but they arise from the light sector and are order $(a\Lambda_{\rm QCD})^n$. Lattice artifacts from the heavy quarks are absorbed into the coefficients, which deviate from their continuum limits; they can be quantified by computing the coefficients, say in perturbation theory in g_0^2, and using tools of the HQET to propagate them to matrix elements.

Through order $1/m_Q^2$ the action of the HQET for lattice QCD can be written

$$S = \int d^4x \left[\bar{h}(D_t + m_1)h - \mathcal{L}^{(1)} - \mathcal{L}^{(2)}\right], \tag{6}$$

where $h = \gamma_0 h$ is the heavy-quark field of the usual HQET, and all interactions $\mathcal{L}^{(n)}$ have the Dirac structure 1, γ_0, or $\boldsymbol{\Sigma}$. The higher-dimension interactions are

$$\mathcal{L}^{(1)} = \frac{\bar{h}\boldsymbol{D}^2 h}{2m_2} + \frac{i\bar{h}\boldsymbol{\Sigma}\cdot\boldsymbol{B}h}{2m_B} \tag{7}$$

$$\mathcal{L}^{(2)} = \frac{\bar{h}[\boldsymbol{\gamma}\cdot\boldsymbol{D}, \boldsymbol{\gamma}\cdot\boldsymbol{E}]h}{8m_E^2} + O(g_0^2), \tag{8}$$

and there are more at even higher dimensions. For example, at dimension 7 one finds $\mathcal{L}^{(3)} = \cdots + w_4 \bar{h} \sum_{i=1}^{3} D_i^4 h$. Rotational invariance of continuum QCD implies $w_4 = 0$ in the usual HQET. In the HQET describing lattice QCD, however, w_4 does not vanish unless the lattice action has been improved accordingly.

To describe matrix elements of (the lattice theory's) currents in the (the lattice theory's) HQET, one must introduce effective currents. For the vector current, for example,

$$Z_{V^{cb}} V_\mu^{cb} \mapsto \eta_V \bar{h}'\gamma_\mu h + V_\mu^{(1)} + V_\mu^{(1,1)} + V_\mu^{(2)}, \tag{9}$$

where

$$V_\mu^{(1)} = \frac{(\boldsymbol{D}\bar{h}')\cdot\boldsymbol{\gamma}\gamma_\mu h}{2m_{3c}} - \frac{\bar{h}'\gamma_\mu\boldsymbol{\gamma}\cdot\boldsymbol{D}h}{2m_{3b}} \tag{10}$$

$$V_\mu^{(1,1)} = -C_V^{(1,1)} \frac{(\boldsymbol{D}\bar{h}')\cdot\boldsymbol{\gamma}\gamma_\mu\boldsymbol{\gamma}\cdot\boldsymbol{D}h}{4m_{3c}m_{3b}} \tag{11}$$

$$V_\mu^{(2)} = \frac{(\boldsymbol{D}^2\bar{h}')\gamma_\mu h}{8m_{D_\perp^2 c}^2} + \frac{i\bar{h}'\boldsymbol{\Sigma}\cdot\boldsymbol{B}\gamma_\mu h}{8m_{\sigma Bc}^2} + \frac{\bar{h}'\boldsymbol{\alpha}\cdot\boldsymbol{E}\gamma_\mu h}{4m_{\alpha Ec}^2} \tag{12}$$

In $V_\mu^{(1,1)}$, the coefficient $C_V^{(1,1)} = 1 + O(\alpha)$. In $V_\mu^{(2)}$, the $1/m_b^2$ terms are not written out, but they should be clear from the $1/m_c^2$ terms.

The basis of operators used in (7)–(12) is not used in all papers on the usual HQET. Because it avoids operators that "vanish by the equations of motion," it is convenient for computing the radiative corrections with the method of sect. 3. Other bases in the literature are related to this one by field redefinitions.

The rest mass m_1 and the inverse "masses" $1/m_2$, $1/m_B$, $1/m_3$, $1/m_E^2$, etc., are the modified coefficients. They depend both on couplings of the lattice action, notably on the bare quark mass and the gauge coupling. The first two coefficients m_1 and $1/m_2$ are known to one loop [5].

In the asymptotic continuum limit, $m_Q a \to 0$, all coefficients obtain the same value as in the usual HQET. In practice, however, everyone's Monte Carlo calculation falls roughly in the range

$$\tfrac{1}{2} \lesssim m_b a \lesssim 2, \tag{13}$$

which is not, in any sense, asymptotic. The point of the modified HQET is that in the range (13) it is ideally suited for propagating the heavy quarks' discretization effects to hadronic matrix elements.

The rest mass m_1 does not propagate to observables for a simple reason. In the Hamiltonian formalism of the effective theory, the rest mass operator $m_1\bar{h}h$ commutes with all other terms in the Hamiltonian. Eigenstates of the HQET are independent of m_1, and mass dependence of the full eigenstates is acquired only from $\mathcal{L}^{(n)}$.

The power corrections to the symmetry limit the matrix elements in (1)–(3) are computed by treating the $\mathcal{L}^{(n)}$ as perturbations. Through order $1/m_Q^2$ one must consider $\bar{h}'\gamma_\mu h$, $V_\mu^{(1)}$, $T\{V_\mu^{(1)}\mathcal{L}^{(1)}\}$, $V_\mu^{(1,1)}$, $T\{\mathcal{L}^{(1)}\bar{h}'\gamma_\mu h\mathcal{L}^{(1)}\}$, $V_\mu^{(2)}$, and $T\{\bar{h}'\gamma_\mu h\mathcal{L}^{(2)}\}$, where T is the time-ordering symbol. The correction $T\{\bar{h}'\gamma_\mu h\mathcal{L}^{(1)}\}$ vanishes by Luke's theorem [6]. One must now repeat, in the present basis of operators, calculations in Refs. [7, 8], keeping track of all the inverse masses [9]. The full results will be presented elsewhere.

Here we give a simple argument why the terms $V_\mu^{(2)}$ and $T\{\bar{h}'\gamma_\mu h\mathcal{L}^{(2)}\}$ drop out of the double ratios. In the HQET's normalization, currents are 1 plus corrections. To order $1/m_Q^2$, the offending terms factor. Taking $T\{\bar{h}'\gamma_\mu h\mathcal{L}^{(2)}\}$ first

$$\langle D|J|B\rangle = (1+\Theta_c/m_{Ec}^2)(1+\Theta_b/m_{Eb}^2), \tag{14}$$

where Θ_c and Θ_b are unknowns. As expected, $T\{\bar{h}'\gamma_\mu h\mathcal{L}^{(2)}\}$, and similarly $V_\mu^{(2)}$, *does* effect the individual matrix elements, but after inserting (14) into (1) or (3) they cancel.

The only effects of order $1/m_Q^2$ which survive are of the form

$$\langle D|J|B\rangle = 1 + \lambda/(m_c m_b), \tag{15}$$

for matrix elements, becoming

$$R = 1 - \lambda(1/m_c - 1/m_b)^2 \tag{16}$$

in (1) and (3). In R_+, R_1, and R_A these effects arise from $V_\mu^{(1,1)}$ and $T\{\mathcal{L}^{(1)}\bar{h}'\gamma_\mu h\mathcal{L}^{(1)}\}$. Neglecting the radiative correction $C_J^{(1,1)}-1$, these double ratios—and so h_+, h_1, and h_{A_1}—have the right mass dependence if $m_2 = m_B = m_3$.

The terms $V_\mu^{(1)}$ and $T\{V_\mu^{(1)}\mathcal{L}^{(1)}\}$ make contributions only to $\langle D|V_i|B\rangle$, so (2) gives h_- the right mass dependence also, if $m_3 = m_2 = m_B$.

3. RADIATIVE CORRECTIONS

To compute the radiative corrections in perturbation theory, one calculates matrix elements of quark states. Taking into account Gordon identities (of the lattice spinors)

$$\begin{aligned} &Z_{V^{cb}}\langle c,\boldsymbol{p}'|V^\mu|b,\boldsymbol{p}\rangle = \bar{u}'\gamma^\mu u\,F_\mu + \\ &\bar{u}'i\sigma^{\mu\nu}u[i(v'-v)_\nu H_{\mu\nu}^{(+)} + i(v'+v)_\nu H_{\mu\nu}^{(-)}], \end{aligned} \tag{17}$$

(sum on ν, but not on μ). The velocities satisfy $\not{v}u = iu$ and $v^2 = -1$. The functions F_μ and $H_{\mu\nu}^{(\pm)}$ are evaluated on (lattice) mass shell, and expanded in $\boldsymbol{p}$ and $\boldsymbol{p}'$.

The factor $Z_{V^{cb}}$ is chosen to yield the radiative corrections of the continuum at $\boldsymbol{p}' = \boldsymbol{p} = \boldsymbol{0}$, viz.

$$F_0|_{\text{on shell}} = \eta_V. \tag{18}$$

Similarly, the rotation parameter d_1 [3] should be adjusted so that $F_i = F_0$ on shell.

The factor $Z_{V^{cb}}$ has strong mass dependence, $Z_{V^{cb}} \sim e^{(m_1^c + m_1^b)/2}$, and its (bare) perturbative series has large coefficients from tadpole diagrams. In matching factors ρ needed for the double ratios both vices cancel: in (1) and (3)

$$\rho_{V_0^{cb}} = \frac{Z_{V^{cb}}Z_{V^{bc}}}{Z_{V^{cc}}Z_{V^{bb}}}, \quad \rho_{A^{cb}} = \frac{Z_{A^{cb}}Z_{A^{bc}}}{Z_{A^{cc}}Z_{A^{bb}}}, \tag{19}$$

where $Z_{A^{cb}}$ is defined by a condition similar to (18), and in (2)

$$\rho_{V_i^{cb}} = \frac{F_i(m_c,m_b) + 2H_{i0}^{(-)}(m_c,m_b)}{F_i(m_c,m_c)}. \tag{20}$$

The factors ρ vary smoothly from the continuum limit (where they equal 1) to the static limit. At one loop $\rho - 1$ prove to be small [10].

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